EXERCISE – II HINTS & SOLUTIONS

Sol.1 C,D

Let $\vec{r} = x\hat{i} + y\hat{j} + 3\hat{k}$

So, $x - 2 = \lambda$, $y - 6 = -3\lambda$

for [B], x = 2, y = 3

Since value of λ is not same hence [B] is rejected.

For [C], x = 1, y = 9

Both equation gives $\lambda = -1$

hence [C] is correct.

Since in given equation $\hat{k} = 0$ here [D] is correct.

Sol.2 A,C,D

Let $\vec{a} = \left(\frac{2}{3}, -\frac{2}{3}, \frac{1}{3}\right)$

$|\vec{a}| = 1 \Rightarrow$ [A]

$$\cos\alpha = \frac{\left(\frac{2}{3}, -\frac{2}{3}, \frac{1}{3}\right).(2, -4, -3)}{1.\sqrt{29}}$$

$$= \frac{\frac{4}{3} + \frac{8}{3} - 1}{\sqrt{29}} = \frac{3}{\sqrt{29}}$$

$\vec{a}.(3,2,-2) = 0 \Rightarrow$ [D] is correct Answer.

$$\vec{a} \times (-1,1,-1/2) = \left(\frac{2}{3}, \frac{-2}{3}, \frac{1}{3}\right) \times \left(-1, 1, -\frac{1}{2}\right)$$

$$\frac{2}{3}\hat{k} + \frac{1}{3}\hat{j} - \frac{2}{3}\hat{k} + \frac{\hat{i}}{3} - \frac{\hat{j}}{3} - \frac{\hat{i}}{3} = 0$$

$\Rightarrow$ [C] is correct options.

Sol.3 A,C

$$\vec{c} = \lambda(\hat{a} - \hat{b}) = \frac{\lambda}{9}(1,-7,-2)$$

$$|\vec{c}| = 5\sqrt{6} \Rightarrow |\vec{c}| = \sqrt{\frac{\lambda^2}{81}}(\sqrt{1+49+4}) = 5\sqrt{6}$$

$|\lambda| = 15 \ (1, -7, -2)$

$$\vec{c} = \frac{5}{3}(1,-7,-2) \text{ or } \vec{c} = \frac{5}{3}(-1,7,2)$$

So [A] and [C] are correct Answer.

Sol.4 A,D

$\vec{a} \times (\vec{b} - \vec{c}) = \vec{d} \times (\vec{b} - \vec{c})$

$(\vec{a} - \vec{d}) \times (\vec{b} - \vec{c}) = 0 \Rightarrow (\vec{a} - \vec{d}) \,||\, (\vec{b} - \vec{c})$

Sol.5 A,B

Let $\hat{a} = \hat{i}$, $\hat{b} = \hat{j}$ then $\hat{a} \times \hat{b} = \hat{i} \times \hat{j} = \hat{k}$

Let required vector be $\vec{r}$ and it makes α, β, γ with x, y, z axis then given $\alpha = \beta = \gamma$

$\Rightarrow \cos\alpha = \cos\beta = \cos\gamma$

$\ell^2 + m^2 + n^2 = 1 \Rightarrow \ell = \pm \frac{1}{\sqrt{3}}$

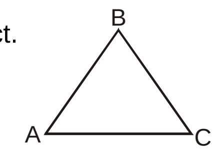
New $\vec{r} = (\ell\hat{i} + m\hat{j} + n\hat{k})$

$$\vec{r} = \pm\frac{1}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k}) \Rightarrow \vec{r} = \pm\frac{1}{\sqrt{3}}(\hat{a} + \hat{b} + \hat{a} \times \hat{b})$$

Sol.6 A,B,C,D

$$G = \frac{\vec{A} + \vec{B} + \vec{C}}{3} = \frac{(a+b+c)}{3}(\hat{i} + \hat{j} + \hat{k})$$

then [A] and [B] are correct.

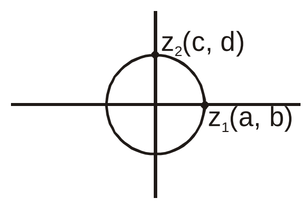


$|\overrightarrow{AB}| - |\overrightarrow{BC}| = |\overrightarrow{CA}|$

then [D] is also correct.

[C] $\Rightarrow$ (By observation)

Sol.7 A,B,C



c = 0

b = 0

$|\vec{z}_1| = \sqrt{a^2 + b^2} = a = r$

$|\vec{z}_2| = \sqrt{c^2 + d^2} = d = r$

$|\vec{w}_1| = \sqrt{a^2 + c^2} = a = r$

$|\vec{w}_2| = \sqrt{b^2 + d^2} = d = r$

$\vec{w}_1;\vec{w}_2$ = ab + cd, then c = 0, b = 0

so $\vec{w}_1;\vec{w}_2 = 0$. Hence A, B, C

Sol.8 B,D

$\vec{r} = (3, 1, -1) + \lambda(2, -1, 2)$

P.V. of 'P' = $3 + 2\lambda$, $1 - \lambda$, $-1 + 2\lambda$

$|\overrightarrow{AP}| = \sqrt{(2\lambda)^2 + (-\lambda)^2 + (2\lambda)^2} = 15$

$\lambda^2 = 25 \Rightarrow \lambda = \pm 5$

for $\lambda = 5 \Rightarrow$ [B] and for $\lambda = -5 \Rightarrow$ [D]

Sol.9 A,B

For a triangle

(A) $\vec{a} + \vec{b} + x\vec{a} - y\vec{b} = 0$

$\Rightarrow$ x = –1, y = 1

$|\vec{a} + \vec{b}| = \sqrt{1 + 1 + 2\cos\theta}$

$= 2\cos\frac{\theta}{2}$

(B) $\dfrac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|} + |\vec{a}+\vec{b}|\left\{\dfrac{\vec{a}-(\vec{a}+\vec{b})}{|\vec{a}||\vec{a}+\vec{b}|}\right\}$

$\vec{a}\cdot\vec{b} - \vec{a}\cdot(\vec{a}+\vec{b}) \quad = -|\vec{a}|^2 = -1$

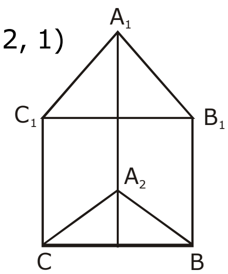
Sol.10 A,D

$\vec{n} = \overrightarrow{AB} \times \overrightarrow{\theta C} = (1, 2, 1)$

$\vec{A} = (1, 0, 1)$

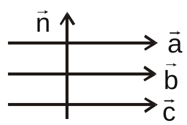
$\vec{A_1} = (0, -2, 0)$

similarly $\vec{A_1} = (2, 2, 2)$



Sol.11 A,C,D

[A]



$\Rightarrow \vec{a},\vec{b},\vec{c}$ are coplanar hence $[\vec{a}\ \vec{b}\ \vec{c}] = 0$

[B] $\ell^2 + m^2 + n^2 = 1$

$n^2 = 1 - \dfrac{3}{4} - \dfrac{1}{2}$

$n^2 = \dfrac{1}{4} - \dfrac{1}{2} = -\dfrac{1}{4}$ (not possible)

Hence false, [C] by observation

[D] same as in options [C] of Q.6

Sol.12 A,C,D

$\vec{a}.\vec{n} = d$

$1 + 10 - 5 = 6 = d$

$2 + 2 - 4 = 0$

Hence line lies in plane $\Rightarrow$ [B] is true.

Sol.13 A,D

$(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d}) = \left(\dfrac{1}{6}, -\dfrac{1}{3}, \dfrac{1}{3}\right)$

$(|\vec{a}||\vec{b}|\sin 30°\hat{\eta})\times(\vec{c}\times\vec{d}) = \left(\dfrac{1}{6}, -\dfrac{1}{3}, \dfrac{1}{3}\right)$

$(\hat{\eta}.\vec{d})\vec{c} - (\hat{\eta}.\vec{c})\vec{d} = \left(\dfrac{1}{3}, -\dfrac{2}{3}, \dfrac{2}{3}\right)$

$\vec{c} - 0 = \dfrac{(1, -2, 2)}{3}$

$\vec{c} = \dfrac{(\hat{i} - 2\hat{j} + 2\hat{k})}{3}$ = [A] and obviously [D will also be the answer as there can be two unit vectors perpendicular to a plane.

Sol.14 A,D

$[\vec{p}\ \ \vec{q}\ \ \vec{r}] = \begin{vmatrix} 2 & 3 & -a \\ b & 5 & -1 \\ 1 & 1 & 3 \end{vmatrix} = 0$ ..... (1)

and $\vec{p}.\vec{q} = 20 \Rightarrow a = 5 - 2b$ ......(2)

from equation (1) : $5a - 9b - ab + 2q = 0$

put $a = 5 - 2b$,

$b = 3, 9$

from equation (2) : ($a = -1$ and -13)

Sol.15 B,D

$\vec{b}.\vec{c} = 0$

$\tan^2\alpha + \tan\alpha - 6 = 0$

$\tan\alpha = 3$ and $\tan\alpha = -2$

Also, $\vec{a}$. (0, 0, 1) < 0

$\sin 2\alpha < 0 \Rightarrow x < 0$

$\sin\alpha\cos\alpha < 0$

$\Rightarrow \alpha$ must lie in second and fourth quadrant.

Now, $\tan\alpha = 3$ is rejected. Hence

$\tan\alpha = -2$

$-\tan\alpha = 2 \Rightarrow \tan(\pi - \alpha) = 2 \Rightarrow \tan(2\pi - \alpha) = 2$

so [B] & [D] is correct options.

Sol.16 A,B,C,D

$\vec{m} = \vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c}$

$\vec{m} = (xy + yz + zx)\,((y - z)\hat{i} + (z - x)\hat{j} + (x - y)\hat{k})$

Now, $\vec{m} \times ((y - z), (z - x), (x - y)) = 0 \Rightarrow$ [A]

$\vec{m}$. (1, 1, 1) = 0 $\Rightarrow$ [B]

$\vec{m}$. ((y + z), (z + x), (x + y)) = 0 $\Rightarrow$ [C]

and $\vec{m}$. (x, y, z) = 0 $\Rightarrow$ [D]

Hence A, B, C, D

Sol.17 A,B,C

$\vec{a}\times b = \vec{c} \Rightarrow \vec{a}.(\vec{b}\times\vec{c}) = \vec{a}\times\vec{c}$

$\Rightarrow \vec{a}.\vec{c} = 0$, $\vec{b}.\vec{c} = 0$

Now, $(\vec{b}\times\vec{c}) = \vec{a}$

$\vec{b}.\vec{a} = 0$, $\vec{c}.\vec{a} = 0$, hence [A],

$(\vec{a}\times\vec{b}) = \vec{c} \Rightarrow \vec{c}.(\vec{a}\times\vec{b}) = |\vec{c}|$

$\left[\vec{c}\ \ \vec{a}\ \ \vec{b}\right] = \left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right] = |\vec{c}|^2 \Rightarrow$ [C]

$\vec{b}\times\vec{c} = \vec{a} \Rightarrow \vec{a}.(\vec{b}\times\vec{c}) = |\vec{a}|^2$

$\left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right] = |\vec{a}|^2 \Rightarrow$ [B]